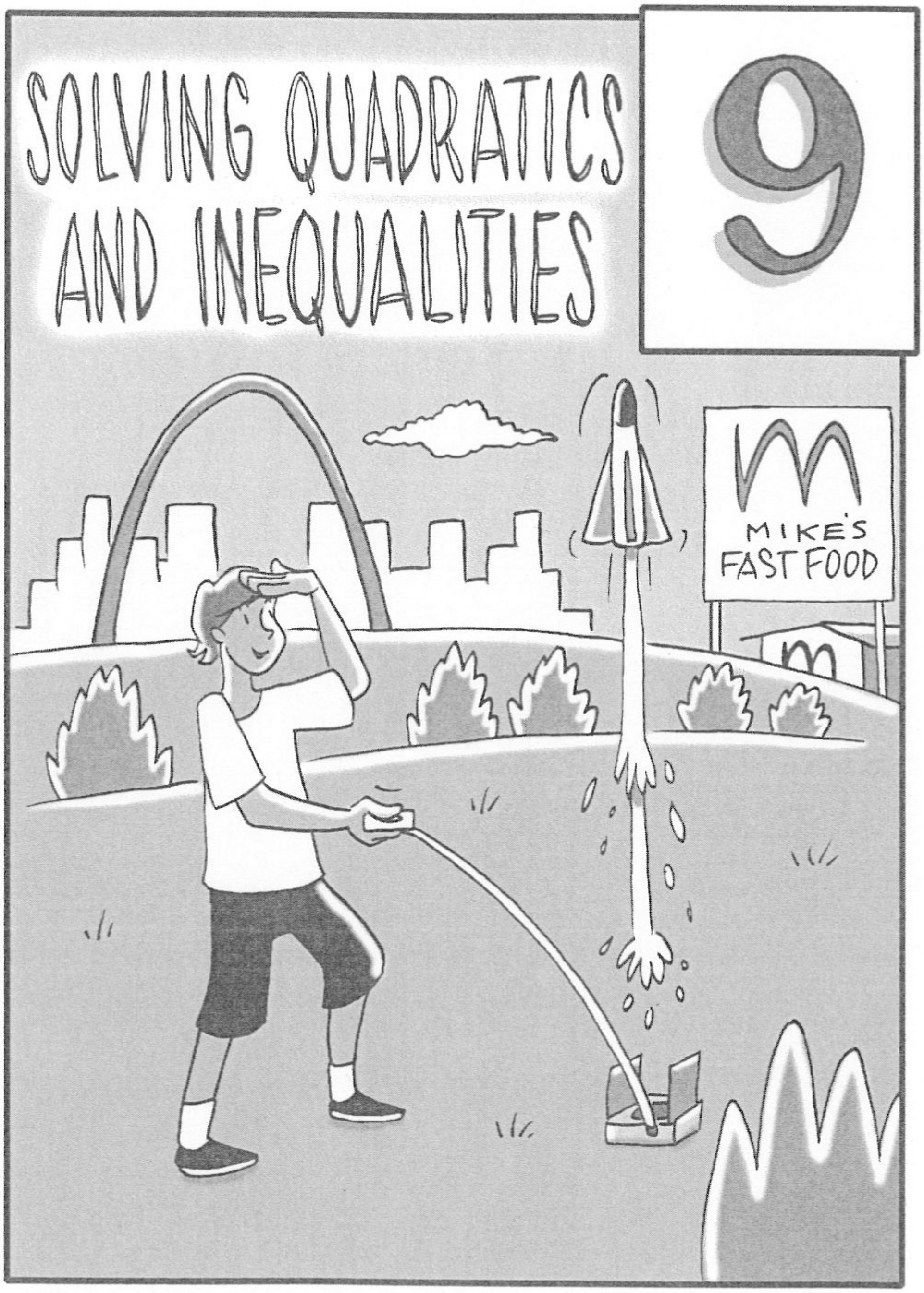
Core Connections Algebra 1



CHAPTER 9 Solving Quadratics and Inequalities

You will start this chapter by extending your ability to solve quadratic equations, and deciding which method of solving is most efficient.

So far in this course you have focused on what you can determine when two expressions are equal. By using what you know about balancing equations, you can now solve linear and quadratic equations for a given variable.

However, what if the two expressions are not equal? If you know that one expression is always larger than the other, what does that tell you about the variable? In this chapter you will learn how to deal with these types of relationships, called *inequalities*. You will develop ways to represent solutions to inequalities both algebraically and graphically.

Guiding Question

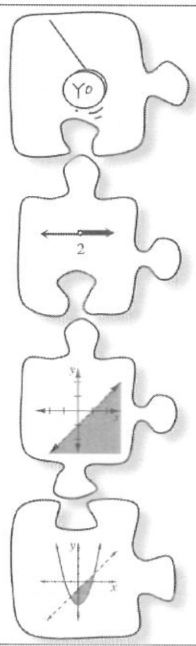
Mathematically proficient students look for and make use of structure.

As you work through this chapter, ask yourself:

Can I look closely to see a pattern or structure in these functions?

In addition, you will extend your ability to work with mathematical sentences by learning how to write inequalities that describe situations.

Chapter Outline



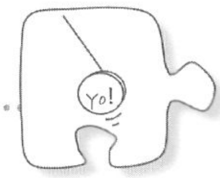
Section 9.1 In this section, you will solve quadratic equations using the Quadratic Formula.

Section 9.2 In this section, you will study how to solve linear inequalities and apply this understanding to solving applications.

Section 9.3 After learning how to represent solutions to one-variable inequalities on a number line, you will study how to represent the solutions of two-variable inequalities on an $x \rightarrow y$ graph.

Section 9.4 In the final section, you will apply what you know about systems of equations to help find the solutions to a system of inequalities.

9.1.1 What else can I solve?



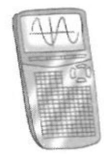
Solving Quadratic Equations

In Chapter 8, you developed a method for finding the x-intercepts of a parabola given by $y = ax^2 + bx + c$. You found the roots or zeros of the quadratic expression by setting it to zero: $ax^2 + bx + c = 0$. Today you will learn how to use that skill to solve a wide variety of quadratic equations.

9-1. Use the Zero Product Property to solve the quadratic equations below.

a. $9 = 3x^2 + 4x - 6$ b. $0 = 3(x-5)(2x+3)$ c. $x^2 + 6x = 0$

9-2. Let's use your calculator to verify your solutions to the original equations in problem 9-1 (before you started rewriting them).



a. Graph $y = 3x^2 + 4x - 6$ with your graphing calculator. Where on the graph would you look to verify your solution to part (a) of problem 9-1?

b. Where in the table on your graphing calculator would you look to verify your solution to part (a) of problem 9-1?

c. Use your graphing calculator to verify your solutions to the other equations in problem 9-1.

9-3. Janelle's team was trying to solve $x^2 - 6x - 1 = 0$.

Janelle suggested using the Zero Product Property.

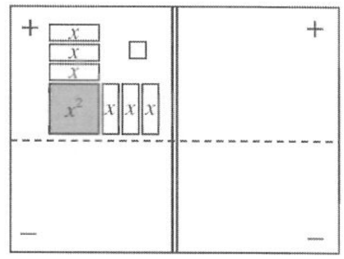
Kira said, *"This quadratic is not factorable. So it doesn't have a solution!"*

a. Is Kira correct? How do you know?

"I know. Let's use the graphing calculator and see if there are solutions." Katelyn said.

b. What did Katelyn mean? If you have not already done so, estimate the solution.

"But," Janelle said, *"the teacher said to get an exact solution. Let's complete the square to help us solve. Here's what the equation would look like."*



$x^2 - 6x - 1$

■ = +1
□ = −1

Erika contributed, *"Oh, I see. Now we can add –8 unit tiles to each side to complete the square."*

"I'm not so sure," said Katelyn. *"That would give us –9 in the corner. Is that right?"*

c. What do you think the teams next legal tile move should be?

d. Complete the square, and solve. Does your solution match your estimate from part (b)?

e. To make sense of the x-intercepts in part (d), you probably wrote your solution as a decimal. Square roots (that are not perfect squares) are **irrational numbers**: they are numbers that cannot be written as a fraction of integers. Irrational numbers cannot be written exactly as a decimal. The decimals continue infinitely and never go into a repeating pattern.

When you write a square root as a decimal you have to round, so it is called **approximate decimal form**. When you use the square root symbol, you are using **exact form** or **radical form**.

Build the equation $x^2 + 4x - 5 = -2$ with algebra tiles. Use Kira's method to complete the square and solve. Write your answer in both exact form and approximate decimal form.

f. Check your answer to part (e) by graphing.

9-4. Solve the following equations using Kira's method. Use generic rectangles to help you complete the square.

a. $x^2 - 2x - 3 = 1$

b. $8x^2 + 40x = 238$

9-5. Use your graphing calculator to verify your solutions to the original equations in problem 9-4.

MATH NOTES

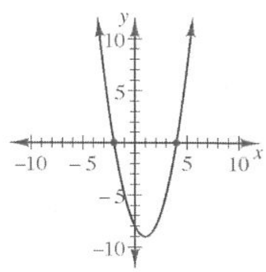
METHODS AND MEANINGS

Zeros and Roots of Quadratics

A **root** or **zero** of a quadratic expression is a value of x that makes the expression equal to zero. For example, the roots or zeros of the quadratic expression $x^2 - 2x - 8$ are the solutions to the equation $x^2 - 2x - 8 = 0$.

The x-intercepts of any quadratic function are roots. You find the x-intercepts by setting the function equal to zero and solving for x.

For example, the quadratic function $f(x) = x^2 - 2x - 8 = (x+2)(x-4)$ is graphed at right. The x-intercepts are at (0, –2) and (0, 4). The roots or zeros are –2 and 4 because the solutions to the equation $x^2 - 2x - 8 = 0$ are –2 and 4.



Review & Preview

9-6. Use your generalized process of completing the square to rewrite and solve each quadratic equation below.

a. $w^2 + 28w + 52 = 0$ b. $x^2 + 5x + 4 = 0$

c. $k^2 - 16k - 17 = 0$ d. $z^2 - 1000z + 60775 = 0$

9-7. For each of the following equations, indicate whether its graph would be a line or a parabola.

a. $5x + 2y = 7$ b. $y = 3x^2$

c. $y = 3$ d. $4x^2 + 3x = 7 + y$

9-8. **Multiple Choice:** Which equations below are equivalent to:

$$\tfrac{1}{2}(6x - 14) + 5x = 2 - 3x + 8\ ?$$

a. $3x - 7 + 5x = 10 - 3x$ b. $3x - 14 + 5x = 2 - 3x + 8$

c. $8x - 14 = 10 - 3x$ d. $6x - 14 + 10x = 4 - 6x + 16$

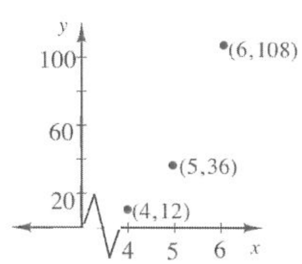
9-9. Rewrite each radical expression in exponent form.

a. $\sqrt[3]{10}$ b. $\sqrt{15}$ c. $\sqrt[4]{18}^{\,3}$ d. $\frac{1}{\sqrt{5}}$

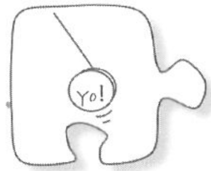
9-10. Examine the two equations below. Where do they intersect?

$$y = 4x - 3$$
$$y = 9x - 13$$

9-11. Find an equation of the sequence represented by this graph.



9.1.2 What if it is not factorable?



Introduction to the Quadratic Formula

In the previous lesson you developed methods to solve quadratic equations. Today you will learn a new method to solve them.

9-12. A quadratic equation can be rewritten in **perfect square form**, $(ax+b)^2=c$, by completing the square.

Write the following equations in perfect square form. Then determine the number of solutions for each quadratic equation. You do not need to actually solve the equations.

Explain how you can quickly determine how many solutions a quadratic equation has once it is written in perfect square form.

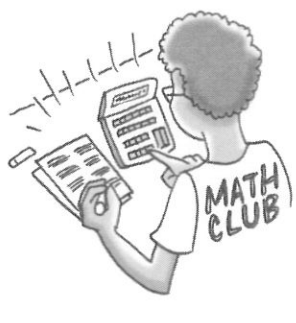
a. $x^2-6x+10=3$

b. $m^2+12m+37=0$

c. $4p^2+36p=-81$

d. $k^2-3k+9=0$

9-13. QUADRATIC FORMULA



One way to calculate the roots of a quadratic equation is by using the **Quadratic Formula**, shown below. This formula uses values a, b, and c from a quadratic equation written in standard form (explained in the next paragraph).

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

a. When the quadratic equation is written in **standard form** (i.e., it looks like $ax^2+bx+c=0$), then a is the number of x^2 terms, b is the number of x-terms, and c is the constant. If $x^2-3x-7=0$, then what are a, b, and c?

Problem continues on next page. →

9-13. *Problem continued from previous page.*

b. The Quadratic Formula calculates *two* possible answers by using the "±" symbol. This symbol (read as "plus or minus") is shorthand notation that tells you to calculate the formula twice: once with addition and once with subtraction in the numerator. Therefore, every Quadratic Formula problem is really two different problems unless the value of $\sqrt{b^2-4ac}$ is 0.

$$\pm$$

$$x=\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{or} \quad x=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

Carefully substitute a, b, and c from $x^2-3x-7=0$ into the Quadratic Formula. Simplify each expression (once using addition and once using subtraction) to solve for x. Write your solution for x in both exact form and approximate form.

c. Make a graph with your graphing calculator to verify your solution.

9-14. The Quadratic Formula is only one of the tools you can use to solve quadratic equations.

a. What are the other methods that you can use?

b. You may be thinking, "Where did this formula come from? Why does it work?" You can find the formula by starting with a generic quadratic $ax^2+bx+c=0$ and using your algebra skills to solve for x. See the Math Notes box for this lesson to learn about one way this formula can be derived. Later, in Chapter 10, you will learn another formal method to derive the Quadratic Formula.

9-15. Use the Quadratic Formula to solve the equations below for x, if possible. Check your solutions with your graphing calculator.

a. $3x^2+7x+2=0$

b. $2x^2-9x-35=0$

c. $8x^2+10x+3=0$

d. $x^2-5x+9=0$

9-16. In your Learning Log, describe how to use the Quadratic Formula. Be sure to include an example. Title this entry "Quadratic Formula" and include today's date.

MATH NOTES

METHODS AND MEANINGS

The Quadratic Formula

Why is $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ a solution of $ax^2+bx+c=0$? One way to derive this formula is shown below.

1. Begin with the quadratic equation in standard form. $$ax^2+bx+c=0$$
2. Multiply each side by $4a$. $$4a(ax^2+bx+c)=4a(0)$$ $$4a^2x^2+4abx+4ac=0$$
3. Add b^2-4ac to each side in order to get a factorable quadratic on the left. $$4a^2x^2+4abx+b^2=b^2-4ac$$
4. The left side can be factored as $(2ax+b)^2$, which is demonstrated in the generic rectangle shown at right. $$(2ax+b)^2=b^2-4ac$$

b	$2abx$	b^2
$2ax$	$4a^2x^2$	$2abx$
	$2ax$	b

5. Take the square root of each side. Since a square root refers to the *positive* root, the absolute value of $2ax+b$ is used. Then by "looking inside" there are two possible values for $2ax+b$: $+\sqrt{b^2-4ac}$ and $-\sqrt{b^2-4ac}$. $$|2ax+b|=\sqrt{b^2-4ac}$$
6. Now continue to solve for x by subtracting b from both sides and dividing by $2a$. Notice that a cannot equal zero or else you will get an error! However, if $a=0$, then this equation would not be quadratic and you would not use this formula. $$2ax+b=\pm\sqrt{b^2-4ac}$$ $$2ax=-b\pm\sqrt{b^2-4ac}$$ $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Thus, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ are solutions of the equation $ax^2+bx+c=0$.

Review & Preview

9-17. Solve the following quadratic equations by factoring and using the Zero Product Property. Be sure to check your solutions.

a. $x^2-13x+42=0$

b. $0=3x^2+10x-8$

c. $2x^2-10x=0$

d. $4x^2+8x-60=0$

9-18. Use the Quadratic Formula to solve $x^2 - 13x + 42 = 0$. Did your solution match the solution from part (a) of problem 9-17?

9-19. Does a quadratic equation always have two solutions? That is, does a parabola always intersect the x-axis twice?

a. If possible, draw an example of a parabola that only intersects the x-axis once.

b. What does it mean if the quadratic equation has no solution? Draw a possible parabola that would cause this to happen.

9-20. Find the equation of the line through the point (–2, 8) with slope $\frac{1}{2}$.

9-21. For each quadratic function below, use the idea of completing the square to write it in graphing form. Then state the vertex of each parabola.

a. $f(x) = x^2 + 4x + 5$

b. $f(x) = x^2 - 6x$

c. In both graphs, does the vertex represent the maximum or minimum value of the parabola?

9-22. The increased demand for vegetarian meals has caused an increase in the price of tofu. If the current cost of $2.99 per pound is increasing 6% per year, what will it cost in 5 years?

9-23. Decide if the statements below are true or false. If necessary, review the descriptions for the inequality symbols $<$, $\leq$, $>$, and $\geq$ in the Lesson 9.2.1 Math Notes box.

a. $11 < -13$

b. $5(2) \geq 10$

c. $13 > -3(2-6)$

d. $4 \leq 4$

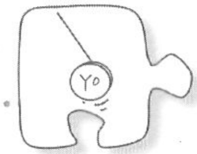
e. $9 \geq -9$

f. $-2 > -2$

g. $-16 < -15$

h. $0 > 6$

9.1.3 What if the equation is not in standard form?



More Solving Quadratic Equations

Today you will apply and extend what you know about solving quadratic equations.

9-24. For the quadratic equation $6x^2+11x-10=0$:

a. Solve it using the Zero Product Property.

b. Solve it using the Quadratic Formula.

c. Did the solutions from parts (a) and (b) match? If not, why not?

9-25. As the Math Notes box from Lesson 9.1.2 demonstrated, the Quadratic Formula can solve any quadratic equation in the form $ax^2+bx+c=0$ if $a \neq 0$. But what if the equation is not in standard form? What if terms are missing? Consider these questions as you solve the quadratic equations below. Share your ideas with your teammates and be prepared to demonstrate your process for the class.

a. $4x^2-121=0$

b. $2x^2-2-3x=0$

c. $15x^2-165x=-630$

d. $36x^2+25=60x$

9-26. THE SAINT LOUIS GATEWAY ARCH

Arches, like the Saint Louis Gateway Arch (pictured at right) are not parabolas. But sometimes arches can be *modeled* by a parabola so that predictions are possible. Suppose the Gateway Arch can be modeled by $y=630-0.00635x^2$, where both x and y represent distances in feet and the origin is the point on the ground directly below the arch's apex (its highest point).

a. Find the x-intercepts of the Gateway Arch. What does this information tell you? Use a calculator to evaluate your answers.

b. How wide is the arch at its base?

c. How tall is the arch? How did you find your solution?

d. Draw a quick sketch of the arch on graph paper, labeling the axes with all of the values you know.

MATH NOTES

METHODS AND MEANINGS

Solving a Quadratic Equation

So far in this course, you have learned three algebraic methods to solve a quadratic equation of the form $ax^2+bx+c=0$.

Example 1: Solve $3x^2+x-14=0$ for x using the Zero Product Property.

Solution: First, factor the quadratic so it is written as a product: $(3x+7)(x-2)=0$. (If factoring is not possible, one of the other methods of solving must be used.) The Zero Product Property states that if the product of two terms is 0, then at least one of the factors must be 0. Thus, $3x+7=0$ or $x-2=0$. Solving these equations for x reveals that $x=-\frac{7}{3}$ or that $x=2$.

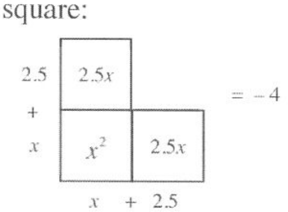
Example 2: Solve $3x^2+x-14=0$ for x using the Quadratic Formula.

Solution: This method works for *any* quadratic. First, identify a, b, and c. a equals the number of x^2 terms, b equals the number of x terms, and c equals the constant. For $3x^2+x-14=0$, $a=3$, $b=1$, and $c=-14$. Substitute the values of a, b, and c into the Quadratic Formula and evaluate the expression twice: once with addition and once with subtraction. Examine this method below:

$$\begin{aligned} x &= \frac{-1+\sqrt{1^2-4(3)(-14)}}{2\cdot 3} \\ &= \frac{-1+\sqrt{169}}{6} \\ &= \frac{12}{6}=2 \end{aligned} \qquad \text{or} \qquad \begin{aligned} x &= \frac{-1-\sqrt{1^2-4(3)(-14)}}{2\cdot 3} \\ &= \frac{-1-\sqrt{169}}{6} \\ &= \frac{-14}{6}=-\frac{7}{3} \end{aligned}$$

Example 3: Solve $x^2+5x+4=0$ by completing the square.

Solution: This method works most efficiently when the coefficient of x^2 is 1. Rewrite the equation as $x^2+5x=-4$. Rewrite the left side as an incomplete square:



Complete the square and rewrite as $(x+2.5)^2-6.25=-4$ or $(x+2.5)^2=2.25$

Take the square root of both sides, $x+2.5=\pm 1.5$. Solving for x reveals that $x=-1$ or $x=-4$.

Review & Preview

9-27. Solve the following quadratic equations by factoring and using the Zero Product Property. Then check your solutions.

a. $x^2 - 10x + 25 = 0$

b. $0 = 3x^2 + 17x - 6$

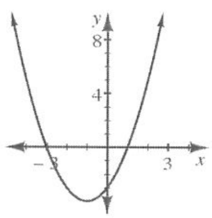
c. $3x^2 - 2x = 5$

d. $16x^2 - 9 = 0$

9-28. Use the Quadratic Formula to solve part (b) of problem 9-27 above. Did your solution match the solution you got by factoring and using the Zero Product Property (in part (b) of problem 9-27)?

9-29. Find the equation that represents the information given below.

a.



b.

x	–4	–3	–2	–1	0	1	2	3	4
y	12	5	0	–3	–4	–3	0	5	12

9-30. Solve the following problem using any method. Write your solution as a sentence.

The length of a rectangle is 5 cm longer than twice the length of the width. If the area of the rectangle is 403 square centimeters, what is the width?

9-31. Which of the points below is a solution to $4x-3y=10$? Note: More than one point may make this equation true.

a. $(1,2)$ b. $(4,2)$ c. $(7,6)$ d. $(4,-3)$

9-32. In order to quickly get people between terminals in the Minneapolis Airport, long "people mover" conveyor belts were installed. Assume that if someone stood still on a conveyor belt, that person would travel 2 feet per second.

a. Since Jung is in a hurry, he decided to walk on the conveyor belt (in the same direction he would travel standing still). If his terminal is 300 feet away and he wants to get there in 60 seconds, how fast does he need to walk with respect to the conveyor belt? (Assume he can ride the conveyor belt the entire distance.)

b. Jacob, who is four years old, decided it would be fun to walk on the conveyor belt in the "wrong" direction (i.e., in the direction opposite to which he would travel if standing still). If he walks for 18 seconds at a rate of 1 foot per second, how far will he travel? In what direction does he travel? Explain.

9-33. Factor each polynomial completely.

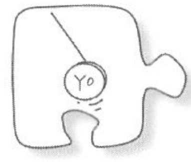
a. $3x^3-3x^2$

b. $2x^2-10x+12$

c. $8x^2-32$

d. $4x^3+10x^2-24x$

9.1.4 Which method should I use?



Choosing a Strategy

You now have more than one algebraic method to solve quadratic equations: using the Zero Product Property, using the Quadratic Formula, and completing the square. How can you decide which strategy is best to try first? By the end of this lesson, you should have some strategies to help you determine which method to try first when solving a quadratic equation.

9-34. Examine the quadratic equations below with your team. For each equation:

- Decide which strategy is best to try first.
- Solve the equation. If your first strategy does not work, switch to another strategy.
- Check your solution(s).

Be prepared to share your process with the class.

a. $x^2+12x+27=0$

b. $0.5x^2+9x-2.8=6$

c. $(3x+4)(2x-1)=0$

d. $x^2+12=8x-4$

e. $x^2+5-2x=0$

f. $20x^2-30x=2x+45$

9-35. LEARNING LOG

With the class, decide when it is best to solve a quadratic by factoring or completing the square, and when you should go directly to the Quadratic Formula. Copy your observations in your Learning Log. Title this entry "Choosing a Strategy to Solve Quadratics" and include today's date.

9-36. While solving $(x-5)(x+2)=-6$, Kyle decided that x must equal 5 or –2. "Not so fast!" exclaimed Stanton, "The product does not equal zero. We need to change the equation first."

a. What is Stanton talking about?

b. How can the equation be rewritten? Discuss this with your team and use your algebraic tools to rewrite the equation so that it can be solved.

c. Solve the resulting equation from part (b) for x.

d. Use the graph and table of $y=(x-5)(x+2)$ on your graphing calculator to check your solution. See whether Kyle's solution matches the solution in part (c).

9-37. MOE'S YO

Moe is playing with a yo-yo. He throws the yo-yo down and then pulls it back up. The motion of the yo-yo is represented by the equation $y=2x^2-4.8x$, where x represents the number of seconds since the yo-yo left Moe's hand, and y represents the vertical height in inches of the yo-yo with respect to Moe's hand. Note that when the yo-yo is in Moe's hand, $y = 0$, and when the yo-yo is below his hand, y is negative.

a. How long is Moe's yo-yo in the air before it comes back to Moe's hand? Write and solve a quadratic equation to find the times that the yo-yo is in Moe's hand.

b. How long does it take for the yo-yo to turn around, that is, to start its return to his hand? Use what you know about parabolas to help you.

c. How long is the yo-yo's string? That is, what is y when the yo-yo changes direction?

d. Draw a sketch of the graph representing the motion of Moe's yo-yo. On the sketch, label the important points: when the yo-yo is in Moe's hand and when it changes direction.

MATH NOTES

METHODS AND MEANINGS

Simplifying Square Roots

Before calculators were universally available, people who wanted to use approximate decimal values for numbers like $\sqrt{45}$ had a few options:

1. Carry around copies of long square-root tables.
2. Use "guess and check" repeatedly to get desired accuracy.
3. "Simplify" the square roots. A square root is simplified when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Simplifying square roots was by far the fastest method. People factored the number as the product of integers hoping to find at least one perfect square number. They memorized approximations of the square roots of the integers from one to ten. Then they could figure out the decimal value by multiplying these memorized facts with the roots of the square numbers. Here are some examples of this method.

Example 1: Simplify $\sqrt{45}$.

First rewrite $\sqrt{45}$ in an equivalent factored form so that one of the factors is a perfect square. Simplify the square root of the perfect square. Verify with your calculator that both $3\sqrt{5}$ and $\sqrt{45}\ \approx 6.71$.

Example 1

$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Examine **Examples 2 and 3** at right. Note that in Example 3, $\sqrt{72}$ was rewritten as $\sqrt{36} \cdot \sqrt{2}$, rather than as $\sqrt{9} \cdot \sqrt{8}$ or $\sqrt{4} \cdot \sqrt{18}$, because 36 is the largest perfect square factor of 72. However, since

Example 2

$$\sqrt{27} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$$

Example 3

$$\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$$

$$\sqrt{4} \cdot \sqrt{18} = 2\sqrt{9 \cdot 2} = 2\sqrt{9} \cdot \sqrt{2} = 2 \cdot 3\sqrt{2} = 6\sqrt{2} \quad \text{and}$$
$$\sqrt{9} \cdot \sqrt{8} = 3\sqrt{4 \cdot 2} = 3\sqrt{4} \cdot \sqrt{2} = 3 \cdot 2\sqrt{2} = 6\sqrt{2}\ ,$$

you can still get the same answer if you simplify it using different methods.

When you take the square root of an integer that is not a perfect square, the result is a decimal that never repeats itself and never ends. It is a number that cannot be written as a fraction using integers. This result is called an **irrational number**. The irrational numbers and the rational numbers together form the **real numbers**.

Generally, since it is now the age of technology, when a **decimal approximation** of an irrational square root is desired, a calculator is used. However for an exact answer, called **exact form** or **radical form**, the number must be written using the $\sqrt{\ }$ symbol.

9-38. Write and solve an equation (or system of equations) for the situation described below. Define your variable(s) and write your solution as a sentence.

Daria has 18 coins that are all nickels and quarters. The number of nickels is 3 more than twice the number of quarters. If she has $1.90 in all, how many nickels does Daria have?

9-39. Solve the following quadratic equations using any method.

a. $10000x^2 - 64 = 0$

b. $9x^2 - 8 = -34x$

c. $2x^2 - 4x + 7 = 0$

d. $3.2x + 0.2x^2 - 5 = 0$

9-40. Find a rule that represents the number of tiles in Figure x for the tile pattern at right.

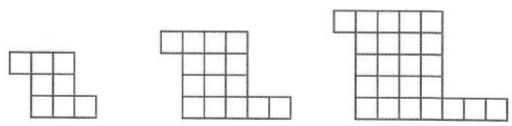


Figure 1 Figure 2 Figure 3

9-41. Solve the equations below for x. Check your solutions.

a. $3x^2 + 3x = 6 + 3x^2$

b. $\frac{5}{x} = \frac{1}{3}$

c. $5 - (2x - 3) = -3x + 6$

d. $6(x - 3) + 2x = 4(2x + 1) - 22$

9-42. Line L passes through the points $(-44, 42)$ and $(-31, 94)$, while line M has the rule $y = 6 + 3x$. Which line is steeper? Justify your answer.

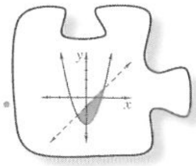
9-43. No Payments For The First Six Months!

You just bought a new tablet computer for $500 including tax. There are no required payments for six months but the company does charge 30% annual interest, compounded monthly, on any unpaid balance.

a. What is the monthly multiplier?

b. If you do not make any payments for six months, how much will you now owe for your new tablet?

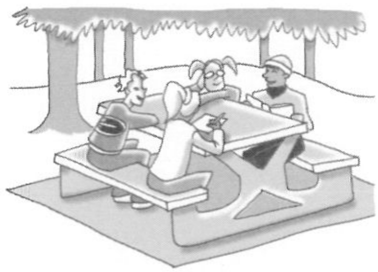
9.4.1 How can I represent it?



Systems of Inequalities

In Chapter 4 you learned that the solution to a system of equations is a point that makes both equations true. But what about the solution of a system of two inequalities? How can you represent these solutions on a graph? How many solutions can a system of inequalities have?

Consider these questions as you learn how to graph a system of inequalities.



9-89. Find your graphs for problem 9-84.

a. Compare your solution graphs for $y \leq -x+5$ and $y > \frac{2}{3}x - 1$ with those of your teammates. Correct any errors. Be sure to focus on whether the boundary line should be included in each graph.

b. What would the graph of the system of inequalities look like? Consider the system of inequalities below. Which points are solutions to this system (that is, which points make *both* inequalities true)?

$$y \leq -x+5$$
$$y > \frac{2}{3}x - 1$$

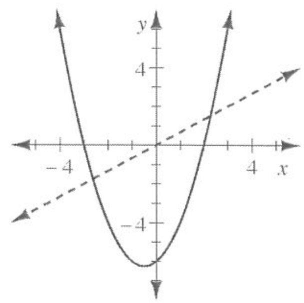
c. If you have not done so already, verify your solution region from part (b) algebraically by substituting the coordinates of a point from your solution region into each inequality.

d. How can you be sure that this region is the only set of points that makes both inequalities true?

9-90. Draw a graph of the region satisfying both constraints below. Start by graphing the boundary lines and then test points to find the region that makes both inequalities true.

$$y < x+2$$
$$y \leq 10 - \frac{3}{4}x$$

9-91. HOW MANY REGIONS?



When graphing the system of inequalities below, Reyna started with the boundary graph of each constraint shown at right.

$$y \le x^2 + x - 6$$
$$y > \tfrac{2}{3}x$$

a. Why is the line dashed while the parabola is not?

b. Find a copy of Reyna's graph on the Lesson 9.4.1B Resource Page provided by your teacher. How many possible solution regions are there? Carefully count each region with your teammates.

c. Pick a point in each region and test it in the system of inequalities. Shade any regions that contain solutions to both inequalities. How many regions make up the solution to this system?

d. Why is (0, 0) not a good point to use to test for this solution?

9-92. How does changing the inequality affect the solution graph? Notice that each system of inequalities below uses the same boundary graphs as Reyna's graph from problem 9-91. However, notice that this time the constraints are slightly altered.

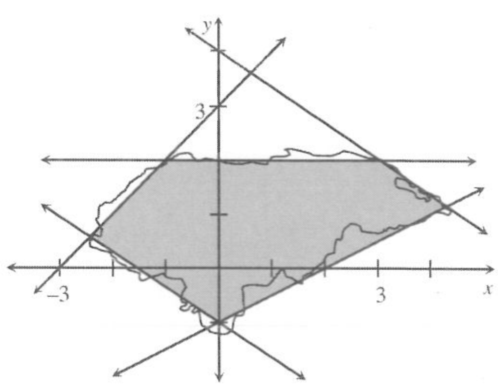
With your teammates, devise a method to determine which region (or regions) are solutions for each system. Shade the appropriate regions on your resource page.

a. $y \ge x^2 + x - 6$, $y > \tfrac{2}{3}x$

b. $y \ge x^2 + x - 6$, $y < \tfrac{2}{3}x$

c. $y \le x^2 + x - 6$, $y < \tfrac{2}{3}x$

9-93. The United Nations asked every nation to write constraints that best approximate its country's shape (the U.N. thinks this will help find each country's area). Honduras sent in its inequalities, but some of the information is unreadable. With your study team, determine the missing parts of the inequalities and rewrite them on your paper.



$y \quad x + 3$ $\qquad y \ge \tfrac{1}{2}x -$

$\quad 2$ $\qquad y \quad -\tfrac{2}{3}x + 4$

$y \quad -\tfrac{2}{3}x - 1$

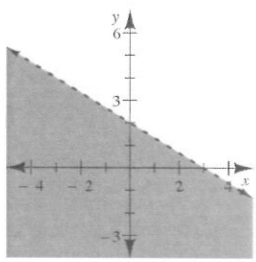
METHODS AND MEANINGS

MATH NOTES

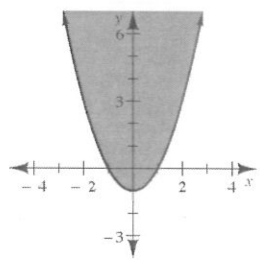
Solving Inequalities with Two Variables

To graph solve an inequality with two variables, first graph the boundary line or curve. If the inequality does not include an equality (that is, if it is $>$ or $<$ rather than $\geq$ or $\leq$), then the graph of the boundary is dashed to indicate that it is not included in the solution. Otherwise, the boundary is a solid line or curve.

Once the boundary is graphed, choose a point that does not lie on the boundary to test in the inequality. If that point makes the inequality true, then the entire region in which that point lies is a solution. Examine the two examples below. There are infinite solutions to each of the inequalities. The shaded portion of the graph is a diagram of all of the solutions.



$y < -\frac{2}{3}x + 2$

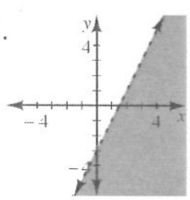


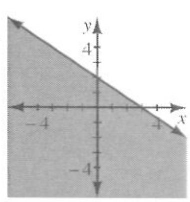
$y \geq x^2 - 1$

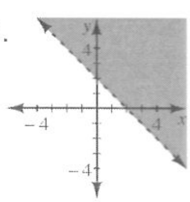
Review & Preview

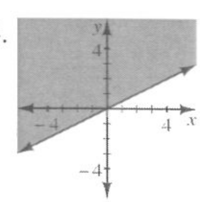
9-94. Match each graph below with the correct inequality.

a. $y > -x + 2$ b. $y < 2x - 3$ c. $y \geq \frac{1}{2}x$ d. $y \leq -\frac{2}{3}x + 2$

1. 

2. 

3. 

4. 

9-95. Solve each inequality below. Represent the solutions on number lines.

a. $7x-2<3+2x$

b. $\frac{1}{3}x \geq 2$

c. $3(2m-1)-5m \leq -1$

d. $2k+3 \leq 2k+1$

9-96. Three years ago the average price of a movie ticket was \$8.75 and now it is \$11.00. What was the annual multiplier and the percent increase?

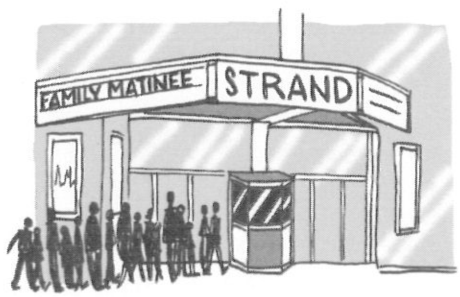
9-97. Factor the following quadratics completely.

a. $5x^3+13x^2-6x$

b. $6t^2-26t+8$

c. $6x^2-24$

9-98. When a family with two adults and three children bought tickets for a movie, they paid a total of \$27.75. The next family in line, with two children and three adults, paid \$32.25 for the same movie. Find the adult and child ticket prices by writing a system of equations with two variables.



9-99. Solve the equation below by completing the square. Give you answer in exact (radical) form.

$$x^2-6x+3=0$$

9-100. **Multiple Choice:** Which of the points below is a solution of $y<|x-3|$?

a. (2, 1)

b. (–4, 5)

c. (–2, 8)

d. (0, 3)

Chapter 9 Closure What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① TEAM BRAINSTORM

What have you studied in this chapter? What ideas were important in what you learned? With your team, brainstorm a list. Be as detailed as you can. To help get you started, a list of Learning Log entries and Math Notes boxes are below.

What topics, ideas, and words that you learned *before* this chapter are connected to the new ideas in this chapter? Again, be as detailed as you can.

How long can you make your list? Challenge yourselves. Be prepared to share your team's ideas with the class.

Learning Log Entries

- Lesson 9.1.2 – Quadratic Formula
- Lesson 9.1.4 – Choosing a Strategy to Solve Quadratics
- Lesson 9.3.1 – Graphing Linear Inequalities
- Lesson 9.4.2 – Graphing Systems of Inequalities

Math Notes

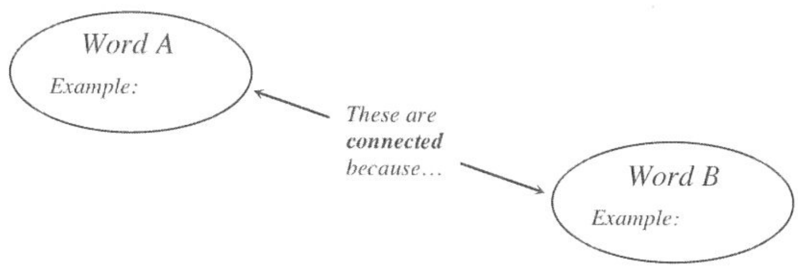
- Lesson 9.1.1 – Zeros and Roots of Quadratics
- Lesson 9.1.2 – The Quadratic Formula
- Lesson 9.1.3 – Solving a Quadratic Equation
- Lesson 9.1.4 – Simplifying Square Roots
- Lesson 9.2.1 – Inequality Symbols
- Lesson 9.3.1 – Curve Fitting an Exponential Function
- Lesson 9.3.2 – Solving One-Variable Linear Inequalities
- Lesson 9.4.1 – Solving Inequalities with Two Variables

② MAKING CONNECTIONS

Below is a list of the vocabulary used in this chapter. Make sure that you are familiar with all of these words and know what they mean. Refer to the glossary or index for any words that you do not yet understand.

Zero Product Property	boundary	coordinates
quadratic equation	graph	inequality
complete the square	region	Quadratic Formula
system of inequalities	standard form	factoring
number line	solution	

Make a concept map showing all of the connections you can find among the key words and ideas listed above. To show a connection between two words, draw a line between them and explain the connection, as shown in the model below. A word can be connected to any other word as long as you can justify the connection. For each key word or idea, provide an example or sketch that shows the idea.



Your teacher may provide you with vocabulary cards to help you get started. If you use the cards to plan your concept map, be sure either to re-draw your concept map on your paper or to glue the vocabulary cards to a poster with all of the connections explained for others to see and understand.

While you are making your map, your team may think of related words or ideas that are not listed here. Be sure to include these ideas on your concept map.

③ PORTFOLIO: EVIDENCE OF MATHEMATICAL PROFICIENCY

Carefully copy your work from problem 9-102, modifying and expanding it if necessary, to showcase your understanding of solving inequalities.

Solve problem 9-37, Moe's Yo, in your portfolio to show your growth in understanding parabolas, including showcasing your new ability to solve quadratic equations.

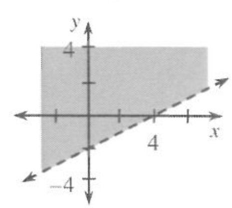
Your teacher may give you the Chapter 9 Closure Resource Page: Quadratic Multiple Representations Graphic Organizer. Complete the Resource Page for Moe's Yo from problem 9-37. Showcase your current understanding of the multiple representations of a quadratic function.

④ WHAT HAVE I LEARNED?

Most of the problems in this section represent typical problems found in this chapter. They serve as a gauge for you. You can use them to determine which types of problems you can do well and which types of problems require further study and practice. Even if your teacher does not assign this section, it is a good idea to try these problems and find out for yourself what you know and what you still need to work on.

Solve each problem as completely as you can. The table at the end of the closure section has answers to these problems. It also tells you where you can find additional help and practice with problems like these.

CL 9-120. Write an inequality that represents the graph at right.



CL 9-121. Is the point (0, 4) a solution to the system of inequalities at right? Justify your answer.

$y \le -3x + 4$
$y > x^2 + 3x - 2$

CL 9-122. Factor these quadratic expressions completely, if possible.

a. $x^2 + x - 30$

b. $-3x^3 + 23x^2 - 14x$

c. $2x^2 - 5x + 4$

d. $6x^3 + 10x^2 - 24x$

CL 9-123. Solve each inequality below for the given variable. Then represent each solution on a number line.

a. $4x - 3 \geq 9$

b. $3(t + 4) < 5$

c. $\frac{2y}{7} < 8$

d. $5x + 4 > -3(x - 8)$

CL 9-124. Brian was holding a ballroom dance. He wanted to make sure girls would come, so he charged boys \$5 to get in but girls only \$3. The 45 people who came paid a total of \$175. How many girls came to the dance?

CL 9-125. Solve each quadratic equation using the specified method.

a. The Quadratic Formula

$0 = 3x^2 + 4x - 7$

b. Factoring

$x^2 - 3x - 18 = 0$

c. Completing the square

$x^2 + 4x + 1 = 0$

d. Using a graph

$2x^2 + 5x - 12 = 0$

CL 9-126. Given the quadratic function $f(x) = (x - 1)^2 - 4$:

a. State the location of the vertex.

b. Determine the x-intercepts.

c. Sketch a graph of the function.

CL 9-127. Graph the system of inequalities below on graph paper.

$$y < x^2$$
$$y \geq x + 2$$

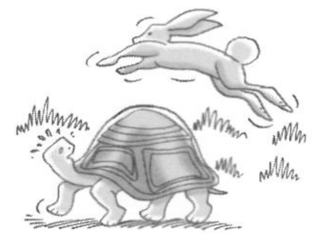
CL 9-128. Lew says to his granddaughter Audrey, "Even if you tripled your age and added 9, you still wouldn't be as old as I am." Lew is 60 years old. Write and solve an inequality to determine the possible ages Audrey could be.

CL 9-129. The cost to rent a DVD has decreased 10% per year over the past several years.

a. If the current cost is \$5, write an exponential equation describing this situation.

b. According to the equation, what did it cost to rent a DVD 5 years ago?

CL 9-130. The hare leaps 500 centimeters every 20 seconds. The tortoise crawls 250 centimeters every 50 seconds, but gets a 1000-centimeter head start. Use any method you know to determine how long it takes the hare to catch up to the tortoise.

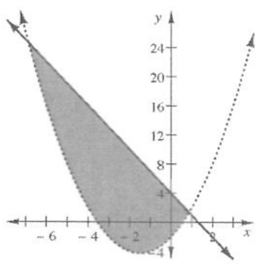
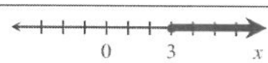
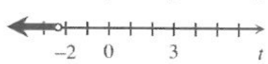
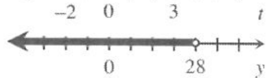
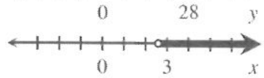


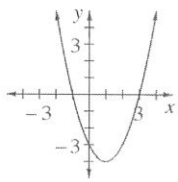
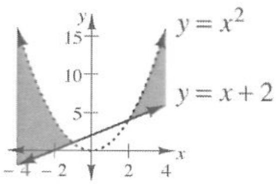
CL 9-131. Find the equation of an exponential function of the form $y = ab^x$ that passes through the points $(3, 13.5)$ and $(5, 30.375)$.

CL 9-132. Check your answers using the table at the end of the closure section. Which problems do you feel confident about? Which problems were hard? Use the table to make a list of topics you need help on and a list of topics you need to practice more.

Answers and Support for Closure Activity #4
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 9-120.	$y > \frac{1}{2}x - 2$	Section 9.3 MN: 9.4.1 LL: 9.3.1	Problems 9-81, 9-84, 9-94, and 9-105
CL 9-121.	Yes, because it makes both inequalities true; the point $(0, 4)$ lies on the solution graph $x^2 + 3x - 2 < y \le -3x + 4$. Therefore, it is a solution to the system of inequalities. 	Section 9.4 LL: 9.4.2	Problems 9-31, 9-100, and 9-106
CL 9-122.	a. $(x+6)(x-5)$ b. $x(-3x+2)(x-7)$ c. not factorable d. $2x(x+3)(3x-4)$	Section 8.1 MN: 8.1.4 LL: 8.1.3 and 8.1.5	Problems CL 8-112, 8-74, 9-33, 9-64, and 9-97
CL 9-123.	a. $x \ge 3$  b. $t < -\frac{7}{3}$  c. $y < 28$  d. $x > 2.5$ 	Section 9.2 MN: 9.2.1 and 9.3.2	Problems 9-51, 9-59, 9-70, 9-95, 9-107, and 9-116
CL 9-124.	Let b = # boys, g = # girls. $5b + 3g = 175$, $b + g = 45$. 25 girls came to the dance.	Chapter 4 Checkpoints 7A and 7B MN: 4.2.3 LL: 4.2.3	Problems CL 7-118, CL 8-118, 9-30, 9-38, 9-50, and 9-98

Problem	Solution	Need Help?	More Practice
CL 9-125.	a. $x=1$ or $x=-\frac{7}{3}$ b. $x=6$ or $x=-3$ c. $x=-2\pm\sqrt{3}\approx-0.27,-3.73$ d. $x\approx-4$, or 1.5	Section 9.1 MN: 9.1.1, 9.1.2, and 9.1.3 LL: 9.1.2 and 9.1.4	Problems 9-6, 9-17, 9-18, 9-27, 9-28, 9-39, 9-52, 9-55, 9-63, 9-99, and 9-117
CL 9-126.	a. $(1,-4)$ b. $x=3,-1$ c. See graph at right. 	Lessons 8.2.3 and 8.2.5 MN: 8.2.4	Problems 9-21, 9-76, and 9-88
CL 9-127.		Sections 9.3 and 9.4 MN: 9.4.1 LL: 9.3.1 and 9.4.2	Problems 9-106 and 9-110
CL 9-128.	$3A+9<60$ $A<17$ Audrey is less than 17 years old.	Section 9.2	Problems 9-71, 9-85, 9-110, and 9-115
CL 9-129.	a. $f(x)=5(0.9)^x$ b. $5(0.9)^{-5}\approx\$8.47$	Checkpoint 9	Problems 8-20, 9-22, 9-43, 9-96, and 9-119
CL 9-130.	The hare catches up to the tortoise after 50 seconds.	Lessons 2.2.2 and 2.2.3	Problems 7-27, 8-41, 9-32, and 9-87
CL 9-131.	$y=4(1.5)^x$	Lessons 7.2.1 and 7.2.2 MN: 9.3.1	Problems CL 7-112 and 9-72